Jumps in an Stochastic Optimization: Self-Financing Portfolio for Risk Nuetral Investors: Does Bequest Matter ?

Şaziye Gazioğlu

M. Kemal University, IIBF, Antakya, Hatay
Department of Economics and
Institute for Applied Mathematics(IAM)
Middle East Technical University (METU), 0653,
Ankara, Turkey
and University of Aberdeen
Dunbar Street, Edward Wright Building
Old Aberdeen,Aberdeen, UK
e-mail: s.gazioglu AT abdn.ac.uk
and

Azize Bastıyalı-Hayfavi

and

Ozge Sezgin

Institute for Applied Mathematics(IAM)
Middle East Technical University(METU) , 0653,
Ankara, Turkey
e-mail: hafavi AT metu.edu.tr

Abstract

We optimized consumer/ investor behaviour, subject to self-financing constraint using stochastic dynamics system with jumps. Our aim in this paper is to compare a stochastic optimization model with and without jump in a self-financing Portfolio Model, for a risk nuetral investors. We also introduce a model with and without bequest to the dynamic system. In this paper, our contributed to the literature is to introduce an analytical solution of the utility maximizing model and investigate the consequences of jumps and bequest to the model. A previous model by Gazioglu Bastiyali-Hafavi (2010) used optimization, only with Brownian motion during optimization. In this paper, our contribution to the literature is to introduce jumps into Poisson process with various intensities. We compare the model with and without jumps for risk neutral investors. Furthermore, we compare the results of the cases with and without bequest as a form of wealth.
Key words: Self-financing Portfolio, Stochastic Optimal Control Problem, Boundary Conditions, Bequest

1 Introduction

Campbell (1993) used intertemporal budget constraint and constructed an asset pricing model without the consumption data. However, since the consumption-wealth ratio is assumed to be constant, overestimation occurred in Euler equation because of perfect correlation between wealth and consumption growth i.e.: $dC_t/C_t = dx_t/x_t = \mu_w dt + v_w dw_t$. They set no restriction in their model for growth of wealth and consumption while optimising the self-financing portfolio model with Brownian motion. They used Hamilton-Jacobi-Bellman (HJB) equation. They investigated two cases : 1. When the terminal wealth is assumed to be zero, there is implicit assumption of no bequest. Gazioglu et al (2010) overcame the problem of the assumption in Hansen and Jagannathan (1991) and Epstein and Zin (1989) in relation to the constant consumption-wealth ratio in the Euler equation. They also introduce inheritance into their model, and compare the model with zero wealth at the end of someone's life ie(no unheritance). They found that the risk neutral investors wealth is accumulated more for the case with bequest. The value function starts with a higher value when there is baquest. In this paper we introduced jumps to the Brownian motion, implying random shocks to the system with jumps. This is a well known mathematical modelling, however we have a unique application to a self-financial strategy of portfolio analysis. This is the main contribution of our paper. The model is solve by HJB, which finds a non-linear equation for w(t) which is solved by iterative method by mathlab. We tend to represent Financial Crisis by jumps to the macroeconomic envirenment. Jumps can be both represented by their number- how often (large), and with their magnitute (hight). We applied both of these to the notion of "'Big jump"' together.

In rest of the paper we set out the model in section 2. In section 3 we discuss the effect of jumps on risk neutral investors by holding low risk return assets. We also compare effects of jumps on the model with and without bequest.

2 Mathematical Model

The asset price dynamics:

We consider a financial market consisting of two assets. One is riskless asset with a price denoted by $S^0(t)$ and the other is the risky asset with a price denoted by $S(t)$. The asset price dynamics are as:

$$dS_0(t) = S_0(t)rdt. \tag{1}$$

$$dS(t) = S(t-)[\alpha dt + \sigma dBt + ydNt] \tag{2}$$

Here, r is the continuously compounded riskless interest rate, α is the drift parameter and σ is the volatility parameter, y is the constant jump size, Bt is the standard Brownian motion and Nt is the poisson process with intensity λ.

The self-financing wealth process x_t has following dynamics:

$$dx_t = w_t(\alpha - r)x_t dt + (rx_t - c_t)dt + w_t\sigma x_t dBt + w_t y x_t dNt \tag{3}$$

The portfolio problem is:

$$
\begin{aligned}
&J(x,t) = \max_{w_t, c_t, 0 \le t \le T} E_{t,x}\{\int_t^T e^{-\rho(s-t)} U_1(c_s, s) ds + e^{-\rho(T-t)} U_2(x_T)\} \\
&s.t. \\
&dx_t = w_t(\alpha - r)x_t dt + (rx_t - c_t)dt + w_t \sigma x_t dBt + w_t y x_t dNt
\end{aligned}
\tag{4}
$$

Where
$x_t = x$ and $c_t = c$,
$U_1(.)$ and $U_2(.)$ are utility functions defined as:

$$U_1(c,t) = \frac{c^{1-\gamma}}{1-\gamma} \tag{5}$$

$$U_1(x) = Kx^{1-\gamma} \tag{6}$$

By the help of optimality principle of stochastic control, Hamilton Jacobi Bellman (HJB) equation for the problem is as:

$$
\begin{aligned}
&\rho J(x,t) = \max_{w,c\, \epsilon A(x)} \{U_1(c,t) + J_t(x,t) + J_x(x,t)(w(\alpha - r)x + (rx - c)) \\
&+\tfrac{1}{2} J_{xx}(x,t) w^2 x^2 \sigma^2 + \lambda(J(x(1+yw),t) - J(x,t))\}
\end{aligned}
\tag{7}
$$

Here, $A(x)$ is the set of all admissible strategies.
When we solve the problem, the optimal weight is found as:

$$w(t) = \frac{1}{\sigma^2 \gamma} \left[(\alpha - r) + \lambda(1 + yw(t))^{-\gamma} y\right] \tag{8}$$

Since it is a non-linear equation for $w(t)$ it can be easily found by iterative numerical methods.

The optimal consumption is found as:

$$
\begin{aligned}
&c(t,x) = \left[-\tfrac{a_1}{b_1} + \left(1 + \tfrac{a_1}{b_1}\right) e^{\frac{b_1}{\gamma}(T-t)}\right]^{-1} K^{-\frac{1}{\gamma}} (1-\gamma)^{-\frac{1}{\gamma}} x \\
&a_1 = [\gamma K^{-\frac{1}{\gamma}} (1-\gamma)^{-\frac{1}{\gamma}}] \\
&b_1 = [w^*(\alpha - r)(1-\gamma) + r(1-\gamma) - \tfrac{1}{2}(1-\gamma)(w^*)^2 \sigma^2 \gamma + \lambda(1 + yw^*)^{1-\gamma} - \lambda - \rho]
\end{aligned}
\tag{9}
$$

The optimal solution implies the following value function

$$J(x,t) = \left[-\frac{a1}{b1} + \left(1 + \frac{a1}{b1}\right) e^{\frac{b1}{\gamma}(T-t)} \right]^{\gamma} K x^{1-\gamma} \tag{10}$$

3 Simulation Results

In this section we will compare envirement with and without jumps in the Brownian motion. The jumps may represents external shocks to the economic envirement as any kind of Financial Crisis. We will compare high and no/low bequest for a risk neutral agents in an envirement with and without jumps. It is important to investiage how risk neutral investors react in consumption to financial crisis. This has implication to macroeconomic and economic growth. We assume plausable parameters for the base run as follows: The low return assets [alpha= 0.5] , the risk neutral agent [gamma= 0.75] with low risk assets [sigma= 0.6]. The power function in the utility function implies that the coefficient of the relative risk aversion (CRRA) and the intertemporal elasticity substitution (IES) , [gamma] are the same. This is the coefficient that determines the curvature of the utility function. Relatively large gamma represents risk averse agents. As gamma approaches to 0 , the agent becomes risk neurtal. We assume small enough gamma to be 0.75. The bequest is represented with K=2, no/small bequest is represented with K=1. Big jumps have two distict elements: a) The Large/small frequency of Jumps is represented by lamda=2,1 and b) The high/Low magnitute of Jumps is represented by y=2,1.

High and No/small Jump Envirenment and With or Without Baquest for the Risk Neutral investors When we compare the case with no jump and with jump, the agents consume less for the jump case, as they are very cautious in their spending. The value function turns to be a convex rather than a concave function for the case with jump. They appear to be risk seeking when there is no jump in the system. However, when there is jump in the system, consumption is half and the value function shows that consumer become risk averse (risk- avoiding). This is despite the nature of the consumption function is risk neutral. This shows that consumer behavior may change according to the dynamics of the economic environment.

THE CASE WITH NO JUMP For the case with No bequest, consumer without jump starts with unit 7 but, since there is no possibility of leaving bequest, they have constant consumption for seven periods they overspend everything at the end of their life-span. Their value function for no bequest is concave downwards which implies that the agents are risk takers in financial terms, at the end of their life time. For the case with bequest, consumers are neither borrower, nor savers The value function becomes steeper at the end of their life time, as their consumption reduces.

CASE WITH BIG JUMP For the case with no bequest and big jumps consumer spend at the end of their life cycle. They are very cautious in their spending. . When there is jump, they spend less overall and they are risk averse in their value function, with convex shape. When there is bequest, consumption is at level 5 for 7 period. The value function of the bequest case is concave

4 Conclusions

In this paper we represented any financial crisis as stochastic shocks with jumps. We investigate a risk neutral agent reacting to jumps, which represents the "'financial crisis"' in the economy. We also investigated possibility of "'bequest"'. Our results indicate that "'bequest"' is important for the environment where, no/ small jumps occurs. However, if there is no jump, with small bequest, consumption steadily decreases. A risk neutral investor has a convex consumption function, with steady increase over time from 16 to 32 units. A risk lover investor would have a concave consumption behavior starting with 66 unit and up with 60 units of consumption.

High jumps create insecurity to the risk neutral and their consumption behavior is convex. This implies that the risk lover investor consumes more between period 1-6, while risk neutral investors have minimum consumption, between 1-6 periods and spends more in the last period. We conclude that frequency of the Financial Crisis and their magnetute influence both risk neutral and risk lover investors in their optimum consumption. As consumption reduces, adverse effect occurs. The result is an economic contraction. We confirm that prevention of these Financial Crisis is important to prevent real sector depression.

5 References

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Appendix

γ*: Coefficient of the Ralative Risk aversion. In power function, it is also equal to intertemporal elasticity substitution.*

δ*: Discovent factor*

σ^2*: Variance (i.e.* σ*: SD (sigma))*

α*: Investment return on the risky assets.*

r: Investment return on the riskless assets.

Case 1: Risk Neutral (Low risk aversion) with low risk return asset (γ) *:*

(γ) *: Low* ($\gamma = 0.8$[1]*), Relatively Low risk averse*
$(\alpha - r)$ *: low (0.5-0.3) risk return assets*
σ *: (0.8)Low Variance .*

$$\delta : timedicounting, , , 0.98$$

(10)

[1]Lower coefficient than 0.8 created problems with the simulation results