Risk Quadrangle and Applications in Day-Trading of Equity Indices

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Agenda

- Introductory words about activities
- Bubbles: Positive and Negative issues
- Fundamental quadrangle working paper of Rockafellar and Uryasev
  www.ise.ufl.edu/uryasev/quadrangle_WP_101111.pdf
  - CVaR optimization
  - Percentile regression
  - Examples of quadrangles
- Library of test problems
  link: http://www.ise.ufl.edu/uryasev/testproblems/
- Hedging strategies for equities
  link: www.aorda.com/aod/static/documents/Protecting_Equity_Investments.pdf

Introductory Words about Activities

- University of Florida webpage: www.ise.ufl.edu/uryasev/
  - Publications
  - Library of Test Problems with data, codes, and solutions

- American Optimal Decisions website
  - www.aorda.com/aod/

- American Optimal Advisors website
  - www.aorda.com/aoa/

Housing Bubble: Positive Aspects

- About 20% of USA GDP is coming from financial industry
  - Credit derivatives and other derivatives were accounting up to 40% of bank profits prior to 2008 (i.e., about 8% of USA GDP was related to derivatives trading)
- Transfer recourses and generation of wealth
  - Beautiful houses and commercial buildings in France, USA,…
- Leveraging (money borrowing): high return on investment
  - Mortgage (root is death, i.e., you pay/leveraged until you are dead)
  - High return on investment of banks and high salaries and taxes
- Top financial engineering approaches
  - CDO (Collateralized Debt Obligations), CLO, …
  - Top mathematics, engineering, databases, management, …

Fundamental Risk Quadrangle

$$\begin{array}{ccccc} & \text{risk } \mathcal{R} & \longleftrightarrow & \mathcal{D} \text{ deviation} & \\ \text{optimization} & \uparrow & \mathcal{S} & \uparrow & \text{estimation} \\ & \text{regret } \mathcal{V} & \longleftrightarrow & \mathcal{E} \text{ error} & \end{array}$$

$\mathcal{R}(X)$ provides a numerical surrogate for the overall hazard in X,
$\mathcal{D}(X)$ measures the "nonconstancy" in X as its uncertainty,
$\mathcal{E}(X)$ measures the "nonzeroness" in X,
$\mathcal{V}(X)$ measures the "regret" in facing the mix of outcomes of X,
$\mathcal{S}(X)$ is the "statistic" associated with X through $\mathcal{E}$ or equivalently $\mathcal{V}$.

General Relationships

Risk versus deviation:

$$\mathcal{D}(X) = \mathcal{R}(X - EX), \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

Deviation from error:

$$\mathcal{D}(X) = \min_C \{\mathcal{E}(X - C)\}$$

Risk from regret:

$$\mathcal{R}(X) = \min_C \{C + \mathcal{V}(X - C)\}$$

Error versus regret:

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

Statistic equivalently from error or regret:

$$\mathcal{S}(X) = \mathrm{argmin}_C \{\mathcal{E}(X - C)\},$$
$$\mathcal{S}(X) = \mathrm{argmin}_C \{C + \mathcal{V}(X - C)\}$$

Mean-Based (St.Dev. Version) Quadrangle

$$\mathcal{S}(X) = EX = \mu(X) = \text{mean}$$

$$\mathcal{R}(X) = EX + \lambda\,\sigma(X) = \text{safety margin tail risk}$$

$$\mathcal{D}(X) = \lambda\,\sigma(X) = \text{standard deviation, scaled}$$

$$\mathcal{V}(X) = EX + \lambda||X||_2 = L^2\text{-regret, scaled}$$

$$\mathcal{E}(X) = \lambda||X||_2 = L^2\text{-error, scaled}$$

General Relationsh

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C\{\,\mathcal{E}(X - C)\,\}, \qquad \mathcal{R}(X) = \min_C\{\,C + \mathcal{V}(X - C)\,\}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C\{\,\mathcal{E}(X - C)\,\} = \operatorname*{argmin}_C\{\,C + \mathcal{V}(X - C)\,\}$$

Mean-Based (Variance Version) Quadrangle

$$\mathcal{S}(X) = EX = \mu(X)$$

$$\mathcal{R}(X) = EX + \lambda\,\sigma^2(X)$$

$$\mathcal{D}(X) = \lambda\,\sigma^2(X)$$

$$\mathcal{V}(X) = EX + \lambda||X||_2^2 = E[v(X)] \ \text{ for } v(x) = x + \lambda x^2$$

$$\mathcal{E}(X) = \lambda||X||_2^2 = E[e(x)] \ \text{ for } e(x) = \lambda x^2$$

General Relationships

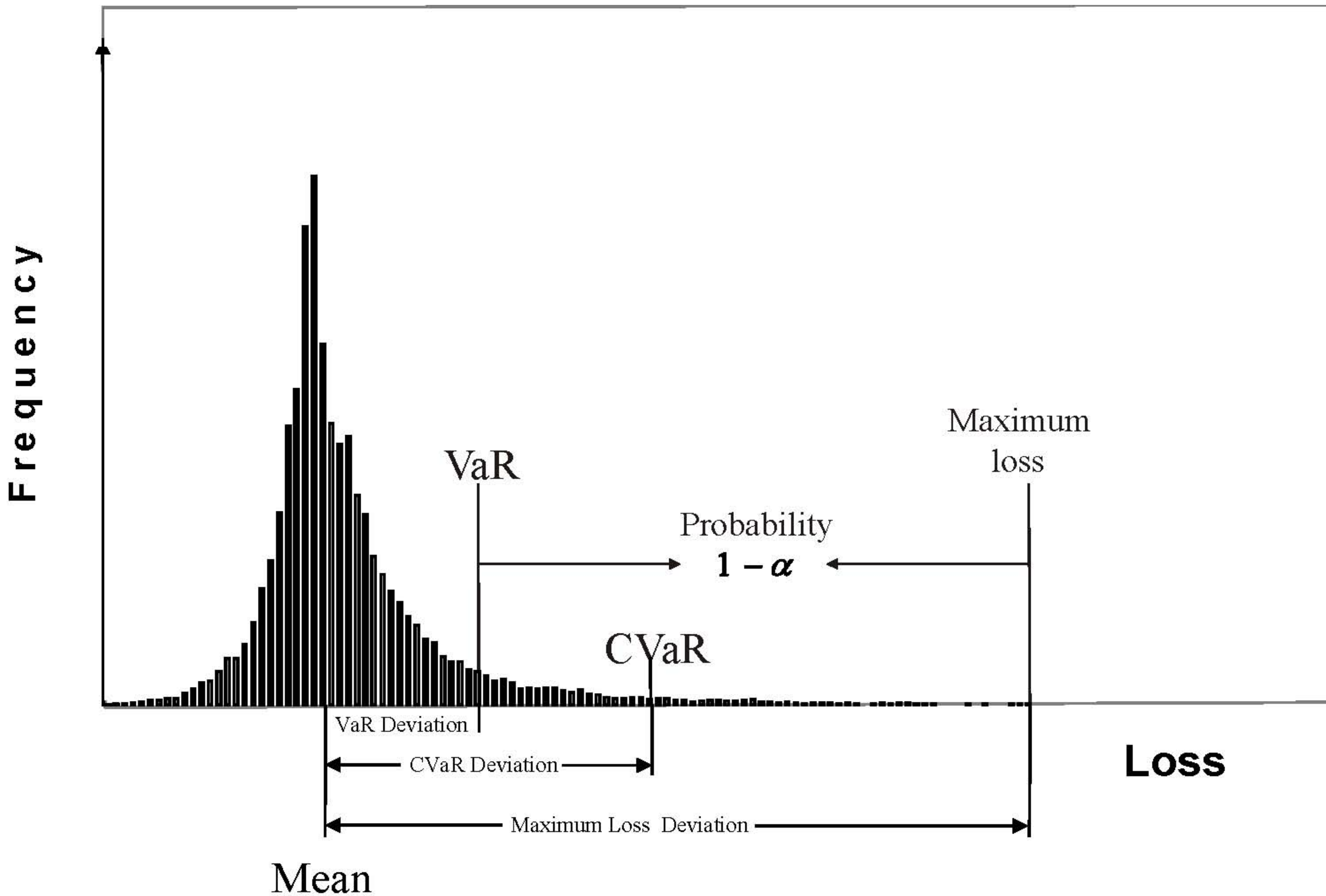
$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C\{\,\mathcal{E}(X-C)\,\}, \qquad \mathcal{R}(X) = \min_C\{\,C + \mathcal{V}(X-C)\,\}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C\{\,\mathcal{E}(X-C)\,\} = \operatorname*{argmin}_C\{\,C + \mathcal{V}(X-C)\,\}$$

VaR and CVaR



Frequency
VaR
Maximum
loss
Probability
$1-\alpha$
CVaR
VaR Deviation
CVaR Deviation
Maximum Loss Deviation
Loss
Mean

Quantile-Based Quadrangle

$\mathcal{S}(X) = \text{VaR}_\alpha(X) = q_\alpha(X) = \text{quantile}$

$\mathcal{R}(X) = \text{CVaR}_\alpha(X) = \overline{q}_\alpha(X) = \text{superquantile}$

$\mathcal{D}(X) = \text{CVaR}_\alpha(X - EX) = \overline{q}_\alpha(X - EX) = \text{superquantile-deviation}$

$\mathcal{V}(X) = \frac{1}{1-\alpha} EX_+ = \text{a penalty expression for regret as scaled average loss}$

$\mathcal{E}(X) = E[\frac{\alpha}{1-\alpha} X_+ + X_-] = \text{normalized Koenker-Bassett error}$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$
$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$
$$\mathcal{D}(X) = \min_C \{ \mathcal{E}(X - C) \}, \qquad \mathcal{R}(X) = \min_C \{ C + \mathcal{V}(X - C) \}$$
$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}$$

VaR vs CVaR in optimization

- VaR is difficult to optimize numerically when losses are not normally distributed
- PSG package allows VaR optimization
- In optimization modeling, CVaR is superior to VaR:
  - For elliptical distribution minimizing VaR, CVaR or Variance is equivalent
  - CVaR can be *expressed as a minimization formula* (Rockafellar and Uryasev, 2000)
  - CVaR preserve convexity

CVaR OPTIMIZATION: MATHEMATICAL BACKGROUND

We want to minimize $CVaR_\alpha(f(x,Y))$

Definition

$$F(x,\zeta) = \zeta + (1-\alpha)^{-1} E(f(x,Y)-\zeta)^+$$

$$= \zeta + \nu \sum_{j=1,J} (f(x,y^j)-\zeta)^+, \text{ in case of equally probable scenarios}$$

$$\nu = ((1-\alpha)J)^{-1} = \text{const}$$

Proposition 1.

$CVaR_\alpha(x) = min_{\zeta\in R} F(x,\zeta)$ and VaR denoted by $\zeta_\alpha(x)$ is a smallest minimizer

Proposition 2.

$$min_{x\in X} CVaR_\alpha(f(x,Y)) = min_{\zeta\in R, x\in X} F(x,\zeta) \qquad (1)$$

- Minimizing of $F(x,\zeta)$ simultaneously calculates VaR= $\zeta_\alpha(x)$, optimal decision x, and optimal CVaR of $f(x,Y)$
- Problem (1) can be reduces to LP using additional variables

CVaR OPTIMIZATION (Cont'd)

- CVaR minimization

$$\min_{\{x \in X\}} \ \text{CVaR}$$

can be reduced to the following linear programming (LP) problem

$$\min_{\{x \in X,\, \zeta \in R,\, z \in R^J\}} \ \zeta + \nu \sum_{\{j=1,\dots,J\}} z_j$$

subject to

$$z_j \geq f(x,y^j) - \zeta, \quad z_j \geq 0, \quad j = 1,\dots J \qquad (\nu = ((1-\alpha)J)^{-1} = \text{const})$$

- By solving LP we find an optimal x* , corresponding VaR, which equals to the lowest optimal ζ *, and minimal CVaR, which equals to the the optimal value of the linear performance function

Stochastic Optimization

- Deterministic setting

$$\text{minimize } f_0(x) \text{ over all } x \in S \subset I\!R^n \text{ subject to } f_i(x) \le 0 \text{ for } i = 1, \dots, m.$$

- Random values depending on decisions variables

$$X_0(x) = \underline{f}_0(x),\ X_1(x) = \underline{f}_1(x), \dots,\ X_m(x) = \underline{f}_m(x)$$

- Stochastic Optimization Problem

$$(\underline{\mathcal{P}}) \qquad \text{minimize } \overline{f}_0(x) = \mathcal{R}_0(\underline{f}_0(x)) \text{ over } x \in S \text{ subject to } \overline{f}_i(x) = \mathcal{R}_i(\underline{f}_i(x)) \le 0, \quad i = 1, \dots, m.$$

Using Quadrangle in Optimization

Regret Theorem. *Consider a stochastic optimization problem* ($\underline{\mathcal{P}}$) *in which each* $\mathcal{R}_i$ *is a regular measure of risk coming from a regular measure of regret* $\mathcal{V}_i$ *with associated statistic* $\mathcal{S}_i$ *by the quadrangle formulas*

$$\mathcal{R}_i(X) = \min_C \{\, C + \mathcal{V}_i(X - C) \,\}, \qquad \mathcal{S}_i(X) = \operatorname*{argmin}_C \{\, C + \mathcal{V}_i(X - C) \,\}. \tag{5.4}$$

Solving ($\underline{\mathcal{P}}$) *can be cast then as solving the expanded problem*

$$(\underline{\mathcal{P}}') \qquad \begin{array}{l} \text{choose } x = (x_1, \ldots, x_n) \text{ and } C_0, C_1, \ldots, C_m \text{ to} \\ \text{minimize } C_0 + \mathcal{V}_0(\underline{f}_0(x) - C_0) \text{ over } x \in S,\ C_i \in I\!R, \\ \text{subject to } C_i + \mathcal{V}_i(\underline{f}_i(x) - C_i) \le 0 \text{ for } i = 1, \ldots, m. \end{array}$$

An optimal solution $(\bar{x}, \bar{C}_0, \bar{C}_1, \ldots, \bar{C}_m)$ *to problem* ($\underline{\mathcal{P}}'$) *provides as* $\bar{x}$ *an optimal solution to problem* ($\underline{\mathcal{P}}$) *and as* $\bar{C}_i$ *a corresponding value of the statistic* $\mathcal{S}_i(\underline{f}_i(\bar{x}))$ *for* $i = 0, 1, \ldots, m$.

Factor Models: Percentile Regression

factors $X_1, \ldots, X_q$ from various sources of information

failure load Y

$Y = c_0 + c_1 X_1 +, \ldots, + c_q X_q + \varepsilon$, where ε is an *error* term

$c_0 + c_1 X_1 +, \ldots, + c_q X_q$ = direct estimator of percentile with α confidence

Percentile regression (Koenker and Basset (1978))

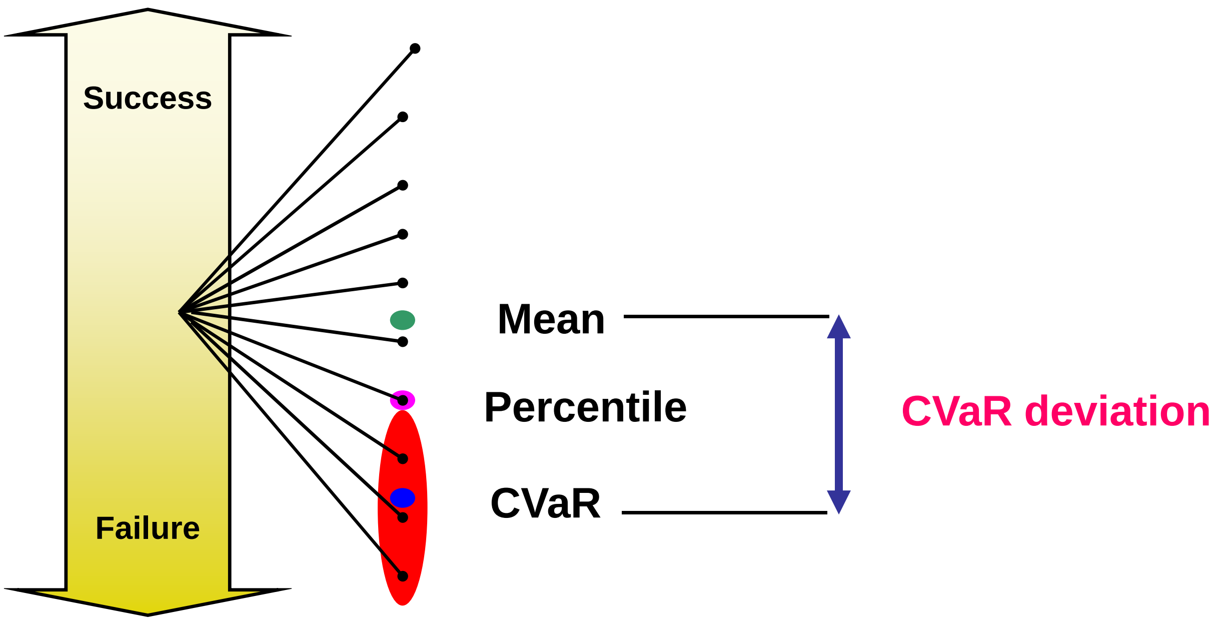
CVaR regression (Rockafellar, Uryasev, Zabarankin (2003))

Percentile Error Function and CVaR Deviation

Statistical approach based on asymmetric percentile error functions: $E[(1-\alpha)(-\varepsilon^{-})+\alpha\varepsilon^{+}]$ **is called <u>Percentile Regression</u>**

ε^{+} **= positive part of error**

ε^{-} **= negative part of error**



Error, Deviation, Statistic

- For the error Koenker and Basset error measure =:

  the corresponding deviation measure < is CVaR deviation

$$D(X) = \min_{C} \mathcal{E}(X - C)$$

  the corresponding statistic K is percentile or VaR

$$S(X) = \operatorname*{argmin}_{C} \mathcal{E}(X - C)$$

- Percentile regression estimates percentile or VaR which is the statistic for the Quantile-based Quadrangle
- Similar results are valid for other quadrangles

Separation Principle

- General regression problem

$$\min_{c_0, c_1, \ldots, c_n} \mathcal{E}(Y - [c_0 + c_1 X_1 + \cdots + c_n X_n])$$

is equivalent to

$$\min_{c_1, \ldots, c\_n} D\,(Y - [\,c_1 X_1 + \cdots + c_n X_n])$$
$$\text{s.t.} \quad c_0 \in S(Y - [\,c_1 X_1 + \cdots + c_n X_n])$$

General Regression Theorem

- Regression problem

$$f(x_1,\ldots,x_n) = C_0 + C_1x_1 + \cdots + C_nx_n$$

$$\text{minimize } \mathcal{E}(Z_f) \text{ over all } f \in \mathcal{C}, \text{ where } Z_f = Y - f(X_1,\ldots,X_n), \tag{5.6}$$

Regression Theorem. *Consider problem (5.6) for random variables $X_1,\ldots,X_n$ and Y in the case of $\mathcal{E}$ being a regular measure of error and $\mathcal{C}$ being a class of functions $f : \mathbb{R}^n \to \mathbb{R}$ such that*

$$f \in \mathcal{C} \implies f + C \in \mathcal{C} \text{ for all } C \in \mathbb{R}. \tag{5.8}$$

Let $\mathcal{D}$ and $\mathcal{S}$ correspond to $\mathcal{E}$ as in the Quadrangle Theorem. Problem (5.6) is equivalent then to the following:

$$\text{minimize } \mathcal{D}(Z_f) \text{ over all } f \in \mathcal{C} \text{ such that } 0 \in \mathcal{S}(Z_f). \tag{5.9}$$

Moreover if $\mathcal{E}$ is of expectation type and $\mathcal{C}$ includes a function f satisfying

$$\begin{aligned} &f(x_1,\ldots,x_n) \in \mathcal{S}(Y(x_1,\ldots,x_n)) \text{ almost surely for } x_1,\ldots,x_n) \in D, \\ &\text{where } Y(x_1,\ldots,x_n) = Y_{X_1=x_1,\ldots,X_n=x_n} \text{ (conditional distribution),} \end{aligned} \tag{5.10}$$

with D being the support of the distribution in $\mathbb{R}^n$ induced by $X_1,\ldots,X_n$, then that f solves the regression problem and tracks this conditional statistic in the sense that

$$f(X_1,\ldots,X_n) = \mathcal{S}(Y \mid X_1,\ldots,X_n) \text{ almost surely.} \tag{5.11}$$

Median-Based Quadrangle

$$\mathcal{S}(X) = \mathrm{VaR}_{1/2}(X) = q_{1/2}(X)$$
$$= \text{median}$$

$$\mathcal{R}(X) = \mathrm{CVaR}_{1/2}(X) = Q_{1/2}(X)$$
$$= \text{“supermedian” (average in tail above median)}$$

$$\mathcal{D}(X) = \mathrm{CVaR}_{1/2}(X - EX) = Q_{1/2}(X - EX)$$
$$= \text{supermedian deviation}$$

$$\mathcal{E}(X) = E|X|$$
$$= \mathcal{L}^1\text{-error}$$

$$\mathcal{V}(X) = 2E[X_+]$$
$$= \mathcal{L}^1\text{-regret}$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$
$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$
$$\mathcal{D}(X) = \min_C \{\, \mathcal{E}(X - C) \,\}, \qquad \mathcal{R}(X) = \min_C \{\, C + \mathcal{V}(X - C) \,\}$$
$$\mathcal{S}(X) = \operatorname*{argmin}_C \{\, \mathcal{E}(X - C) \,\} = \operatorname*{argmin}_C \{\, C + \mathcal{V}(X - C) \,\}$$

Range-Based Quadrangle

$\mathcal{S}(X) = \frac{1}{2}[\sup X + \inf X]$ = center of range of X (if bounded)

$\mathcal{R}(X) = EX + \frac{1}{2}[\sup X - \inf X]$ = range-buffered risk

$\mathcal{D}(X) = \frac{1}{2}[\sup X - \inf X]$ = radius of the range of X (maybe ∞)

$\mathcal{V}(X) = EX + \sup |X| = \mathcal{L}^\infty$-regret

$\mathcal{E}(X) = \sup |X| = \mathcal{L}^\infty$-error

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C \{ \mathcal{E}(X - C) \}, \qquad \mathcal{R}(X) = \min_C \{ C + \mathcal{V}(X - C) \}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}$$

Worst-Case-Based Quadrangle

$\mathcal{S}(X) = \sup X\ =$ top of the range of X (maybe ∞)

$\mathcal{R}(X) = \sup X\ =$ yes, the same as $\mathcal{S}(X)$

$\mathcal{D}(X) = \sup X - EX\ =$ span of the upper range of X (maybe ∞)

$$\mathcal{V}(X) = \begin{cases} 0 & \text{if } X \leq 0 \\ \infty & \text{if } X \not\leq 0 \end{cases} \quad = \text{worst-case-regret}$$

$$\mathcal{E}(X) = \begin{cases} E|X| & \text{if } X \leq 0 \\ \infty & \text{if } X \not\leq 0 \end{cases} \quad = \text{worst-case-error}$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C\{\,\mathcal{E}(X - C)\,\}, \qquad \mathcal{R}(X) = \min_C\{\,C + \mathcal{V}(X - C)\,\}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C\{\,\mathcal{E}(X - C)\,\} = \operatorname*{argmin}_C\{\,C + \mathcal{V}(X - C)\,\}$$

Distributed-Worst-Case-Based Quadrangle

$p_k =$ probability of the kth set of circumstances, with $p_k > 0$, $p_1 + \cdots + p_r = 1$,
$\sup_k X =$ worst of X under circumstances k, for $k = 1, \ldots, r$,
$E_k X =$ conditional expectation of X under circumstances k.

$$\mathcal{S}(X) = p_1 \sup_1 X + \cdots + p_r \sup_r X$$

$$\mathcal{R}(X) = p_1 \sup_1 X + \cdots + p_r \sup_r X \;=\text{ yes, the same as } \mathcal{S}(X)$$

$$\mathcal{D}(X) = p_1[\sup_1 X - E_1 X] + \cdots + p_r[\sup_r X - E_r X]$$

$$\mathcal{V}(X) = \begin{cases} 0 & \text{if } p_1 \sup_1 X + \cdots + p_r \sup_r X \leq 0, \\ \infty & \text{otherwise} \end{cases}$$

$$\mathcal{E}(X) = \begin{cases} E|p_1 E_1 X + \cdots + p_r E_r X| & \text{if } p_1 \sup_1 X + \cdots + p_r \sup_r X \leq 0, \\ \infty & \text{otherwise} \end{cases}$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C \{ \mathcal{E}(X - C) \}, \qquad \mathcal{R}(X) = \min_C \{ C + \mathcal{V}(X - C) \}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}$$

Truncated-Mean-Based Quadrangle

$$T_\beta(x) = \begin{cases} \beta & \text{when } x \geq \beta, \\ x & \text{when } -\beta \leq x \leq \beta, \\ -\beta & \text{when } x \leq -\beta. \end{cases}$$

$$\mathcal{S}(X) = \mu_\beta(X) = \text{value of } C \text{ such that } E[T_\beta(X - C)] = 0$$

$$\mathcal{R}(X) = \mu_\beta(X) + E[v(X - \mu_\beta(X))] \;\; \text{for } v \text{ as below}$$

$$\mathcal{D}(X) = E[e(X - \mu_\beta(X))] \;\; \text{for } e \text{ as below}$$

$$\mathcal{V}(X) = E[v(X)] \;\; \text{with } v(x) = \begin{cases} -\frac{\beta}{2} & \text{when } x \leq -\beta \\ x + \frac{1}{2\beta}x^2 & \text{when } |x| \leq \beta \\ 2x - \frac{\beta}{2} & \text{when } x \geq \beta \end{cases}$$

$$\mathcal{E}(X) = E[e(X)] \;\; \text{with } e(x) = \begin{cases} |x| - \frac{\beta}{2} & \text{when } |x| \geq \beta \\ \frac{1}{2\beta}x^2 & \text{when } |x| \leq \beta \end{cases} \quad \text{Huber-type error}$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C \{\, \mathcal{E}(X - C) \,\}, \qquad \mathcal{R}(X) = \min_C \{\, C + \mathcal{V}(X - C) \,\}$$

$$\mathcal{S}(X) = \underset{C}{\operatorname{argmin}} \{\, \mathcal{E}(X - C) \,\} = \underset{C}{\operatorname{argmin}} \{\, C + \mathcal{V}(X - C) \,\}$$

Log-Exponential-Based Quadrangle

$\mathcal{S}(X) = \log E[\exp X]$ = expression dual to Boltzmann-Shannon entropy

$\mathcal{R}(X) = \log E[\exp X]$ = yes, the same as $\mathcal{S}(X)$

$\mathcal{D}(X) = \log E[\exp(X - EX)]$ = log-exponential deviation

$\mathcal{V}(X) = E[\exp X - 1]$ regret $\quad\longleftrightarrow\quad$ utility $\mathcal{U}(Y) = E[1 - \exp(-Y)]$

$\mathcal{E}(X) = E[\exp X - X - 1]$ = exponential error

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$
$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$
$$\mathcal{D}(X) = \min_C \{\, \mathcal{E}(X - C) \,\}, \qquad \mathcal{R}(X) = \min_C \{\, C + \mathcal{V}(X - C) \,\}$$
$$\mathcal{S}(X) = \operatorname*{argmin}_C \{\, \mathcal{E}(X - C) \,\} = \operatorname*{argmin}_C \{\, C + \mathcal{V}(X - C) \,\}$$

Rate-Based Quadrangle

$$\mathcal{S}(X) = r(X) = \text{ unique } C \geq \sup X - 1 \text{ such that } E\Big[\frac{1}{1-X+C}\Big] = 1$$

$$\mathcal{R}(X) = r(X) + E\Big[\log \frac{1}{1-X+r(X)}\Big]$$

$$\mathcal{D}(X) = r(X) + E\Big[\log \frac{1}{1-X+r(X)} - X\Big]$$

$$\mathcal{V}(X) = E\Big[\log \frac{1}{1-X}\Big] \text{ regret} \quad \longleftrightarrow \quad \text{utility } \mathcal{U}(Y) = E[\log(1+Y)]$$

$$\mathcal{E}(X) = E\Big[\log \frac{1}{1-X} - X\Big]$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$

$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$

$$\mathcal{D}(X) = \min_C \{\, \mathcal{E}(X - C) \,\}, \qquad \mathcal{R}(X) = \min_C \{\, C + \mathcal{V}(X - C) \,\}$$

$$\mathcal{S}(X) = \operatorname*{argmin}_C \{\, \mathcal{E}(X - C) \,\} = \operatorname*{argmin}_C \{\, C + \mathcal{V}(X - C) \,\}$$

Mix-Quantile-Based Quadrangle

$\alpha_i \in (0,1)$ and weights $\lambda_k > 0$, $\sum_{k=1}^{r} \lambda_k = 1$

$\mathcal{S}(X) = \sum_{k=1}^{r} \lambda_k q_{\alpha_k}(X) = \sum_{k=1}^{r} \lambda_k \mathrm{VaR}_{\alpha_k}(X) =$ a mixture[12] of quantiles of X

$\mathcal{R}(X) = \sum_{k=1}^{r} \lambda_k \overline{q}_{\alpha_k}(X) = \sum_{k=1}^{r} \lambda_k \mathrm{CVaR}_{\alpha_k}(X)$
$=$ the corresponding mixture of superquantiles of X

$\mathcal{D}(X) = \sum_{k=1}^{r} \lambda_k \overline{q}_{\alpha_k}(X - EX) = \sum_{k=1}^{r} \lambda_k \mathrm{CVaR}_{\alpha_k}(X - EX)$
$=$ the corresponding mixture of superquantile deviations of X

$$\mathcal{V}(X) = \min_{B_1,\dots,B_r} \left\{ \sum_{k=1}^{r} \lambda_k \mathcal{V}_{\alpha_k}(X - B_k) \,\middle|\, \sum_{k=1}^{r} \lambda_k B_k = 0 \right\}$$
$=$ a derived balance of the regrets $\mathcal{V}_{\alpha_k}(X) = \frac{1}{1-\alpha_k} EX_+$

$$\mathcal{E}(X) = \min_{B_1,\dots,B_r} \left\{ \sum_{k=1}^{r} \lambda_k \mathcal{E}_{\alpha_k}(X - B_k) \,\middle|\, \sum_{k=1}^{r} \lambda_k B_k = 0 \right\}$$
$=$ a derived balance of the errors $\mathcal{E}_{\alpha_k}(X) = E[\frac{\alpha_k}{1-\alpha_k} X_+ + X_-]$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$
$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$
$$\mathcal{D}(X) = \min_C \{ \mathcal{E}(X - C) \}, \qquad \mathcal{R}(X) = \min_C \{ C + \mathcal{V}(X - C) \}$$
$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}$$

Quantile-Radius-Based Quadrangle

$$\mathcal{S}(X) = \tfrac{1}{2}[q_\alpha(X) - q_{1-\alpha}(X)] = \tfrac{1}{2}[\mathrm{VaR}_\alpha(X) - \mathrm{VaR}_{1-\alpha}(X)]$$
$$= \text{the } \alpha\text{-quantile radius of } X, \text{ or } \tfrac{1}{2} \text{ two-tail-VaR}_\alpha \text{ of } X$$

$$\mathcal{R}(X) = EX + \tfrac{1}{2}[\overline{q}_\alpha(X) + \overline{q}_\alpha(-X)] = EX + \tfrac{1}{2}[\mathrm{CVaR}_\alpha(X) + \mathrm{CVaR}_\alpha(-X)]$$
$$= \text{reverted CVaR}_\alpha$$

$$\mathcal{D}(X) = \tfrac{1}{2}[\overline{q}_\alpha(X) + \overline{q}_\alpha(-X)] = \tfrac{1}{2}[\mathrm{CVaR}_\alpha(X) + \mathrm{CVaR}_\alpha(-X)]$$
$$= \text{the } \alpha\text{-superquantile radius of } X$$

$$\mathcal{V}(X) = EX + \min_B \left\{ \tfrac{1}{2(1-\alpha)} E\Big[[B+X]_+ + [B-X]_+ \Big] - B \right\}$$
$$= \alpha\text{-quantile-radius regret in } X$$

$$\mathcal{E}(X) = \tfrac{1}{2(1-\alpha)} \min_B E\Big[[B+X]_+ + [B-X]_+ \Big]$$
$$= \alpha\text{-quantile-radius error in } X$$

General Relationships

$$\mathcal{D}(X) = \mathcal{R}(X) - EX, \qquad \mathcal{R}(X) = EX + \mathcal{D}(X)$$
$$\mathcal{E}(X) = \mathcal{V}(X) - EX, \qquad \mathcal{V}(X) = EX + \mathcal{E}(X)$$
$$\mathcal{D}(X) = \min_C \{ \mathcal{E}(X - C) \}, \qquad \mathcal{R}(X) = \min_C \{ C + \mathcal{V}(X - C) \}$$
$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}$$

Quadrangle Theorem

(a) *The relations* $\mathcal{D}(X) = \mathcal{R}(X) - EX$ *and* $\mathcal{R}(X) = EX + \mathcal{D}(X)$ *give a one-to-one correspondence between regular measures of risk* $\mathcal{R}$ *and regular measures of deviation* $\mathcal{D}$. *In this correspondence,* $\mathcal{R}$ *is positively homogeneous if and only if* $\mathcal{D}$ *is positively homogeneous. On the other hand,*

$$\mathcal{R} \text{ is monotonic if and only if } \mathcal{D}(X) \leq \sup X - EX \text{ for all } X. \tag{3.16}$$

(b) *The relations* $\mathcal{E}(X) = \mathcal{V}(X) - EX$ *and* $\mathcal{V}(X) = EX + \mathcal{E}(X)$ *give a one-to-one correspondence between regular measures of regret* $\mathcal{V}$ *and regular measures of error* $\mathcal{E}$. *In this correspondence,* $\mathcal{V}$ *is positively homogeneous if and only if* $\mathcal{E}$ *is positively homogeneous. On the other hand,*

$$\mathcal{V} \text{ is monotonic if and only if } \mathcal{E}(X) \leq EX \text{ when } X \leq 0. \tag{3.17}$$

(c) *For any regular measure of regret* $\mathcal{V}$, *a regular measure of error* $\mathcal{R}$ *is obtained by*

$$\mathcal{R}(X) = \min_C \Big\{ C + \mathcal{V}(X - C) \Big\}. \tag{3.18}$$

If $\mathcal{V}$ *is positively homogeneous, then* $\mathcal{R}$ *is positively homogeneous, and if* $\mathcal{V}$ *is monotonic, then* $\mathcal{R}$ *is monotonic.*

(d) *For any regular measure of error* $\mathcal{E}$, *a regular measure of deviation* $\mathcal{D}$ *is obtained by*

$$\mathcal{D}(X) = \min_C \Big\{ \mathcal{E}(X - C) \Big\}. \tag{3.19}$$

If $\mathcal{E}$ *is positively homogeneous, then* $\mathcal{D}$ *is positively homogeneous, and if* $\mathcal{E}$ *satisfies the condition in (3.17), then* $\mathcal{D}$ *satisfies the condition in (3.18).*

(e) *In both (c) and (d), as long as the expression being minimized is finite for some* C, *the set of* C *values for which the minimum is attained is a nonempty, closed, bounded interval.*[27] *Moreover when* $\mathcal{V}$ *and* $\mathcal{E}$ *are paired as in (b), the interval comes out the same and gives the associated statistic:*

$$\mathcal{S}(X) = \operatorname*{argmin}_C \{ \mathcal{E}(X - C) \} = \operatorname*{argmin}_C \{ C + \mathcal{V}(X - C) \}. \tag{3.20}$$

Mixing and Scaling Theorems

Mixing Theorem. *For $k = 1, \dots, r$ let $(\mathcal{R}_k, \mathcal{D}_k, \mathcal{V}_k, \mathcal{E}_k)$ be a regular quadrangle quartet with statistic $\mathcal{S}_k$, and consider any weights $\lambda_k > 0$ with $\lambda_1 + \cdots + \lambda_r = 1$. A regular quadrangle quartet $(\mathcal{R}, \mathcal{D}, \mathcal{V}, \mathcal{E})$ with statistic $\mathcal{S}$ is given then by*

$$\begin{aligned}
\mathcal{S}(X) &= \lambda_1 \mathcal{S}_1(X) + \cdots + \lambda_r \mathcal{S}_r(X), \\
\mathcal{R}(X) &= \lambda_1 \mathcal{R}_1(X) + \cdots + \lambda_r \mathcal{R}_r(X), \\
\mathcal{D}(X) &= \lambda_1 \mathcal{D}_1(X) + \cdots + \lambda_r \mathcal{D}_r(X), \\
\mathcal{V}(X) &= \min_{B_1,\dots,B_r} \left\{ \sum_{k=1}^r \lambda_k \mathcal{V}_k(X - B_k) \,\middle|\, \sum_{k=1}^r \lambda_k B_k = 0 \right\}, \\
\mathcal{E}(X) &= \min_{B_1,\dots,B_r} \left\{ \sum_{k=1}^r \lambda_k \mathcal{E}_k(X - B_k) \,\middle|\, \sum_{k=1}^r \lambda_k B_k = 0 \right\}.
\end{aligned} \tag{3.22}$$

Scaling Theorem. *Let $(\mathcal{R}_0, \mathcal{D}_0, \mathcal{V}_0, \mathcal{E}_0)$ be a regular quadrangle quartet with statistic $\mathcal{S}_0$ and consider any $\lambda \in (0, \infty)$. Then a regular quadrangle quartet $(\mathcal{R}, \mathcal{D}, \mathcal{V}, \mathcal{E})$ with statistic $\mathcal{S}$ is given by*

$$\begin{aligned}
\mathcal{S}(X) &= \mathcal{S}_0(X), \\
\mathcal{R}(X) &= (1 - \lambda)EX + \lambda \mathcal{R}_0(X), \\
\mathcal{D}(X) &= \lambda \mathcal{D}_0(X), \\
\mathcal{V}(X) &= (1 - \lambda)EX + \lambda \mathcal{V}_0(X), \\
\mathcal{E}(X) &= \lambda \mathcal{E}_0(X).
\end{aligned} \tag{3.23}$$

Envelope Theorem

Envelope Theorem[58]**.** *The functionals $\mathcal{J}$ that are the conjugates $\mathcal{R}^*$ of the regular measures of risk $\mathcal{R}$ on $\mathcal{L}^2(\Omega)$ are the closed convex functionals $\mathcal{J}$ with effective domains $\mathcal{Q} = \operatorname{dom} \mathcal{J}$ such that*

(a) $EQ = 1$ *for all* $Q \in \mathcal{Q}$,

(b) $0 = \mathcal{J}(1) \leq \mathcal{J}(Q)$ *for all* $Q \in \mathcal{Q}$,

(c) *for each nonconstant* $X \in \mathcal{L}^2(\Omega)$ *there exists* $Q \in \mathcal{Q}$ *such that* $E[XQ] - EX > \mathcal{J}(Q)$.

The dual representation of $\mathcal{R}$ *corresponding to* $\mathcal{J} = \mathcal{R}^*$ *is*

$$\boxed{\mathcal{R}(X) = \sup_{Q \in \mathcal{Q}} \left\{ E[XQ] - \mathcal{J}(Q) \right\}.} \tag{6.8}$$

Here $\mathcal{R}$ *is positively homogeneous if and only if* $\mathcal{J}(Q) = 0$ *for all* $Q \in \mathcal{Q}$, *whereas* $\mathcal{R}$ *is monotonic if and only if* $Q \geq 0$ *for all* $Q \in \mathcal{Q}$.

If $\mathcal{V}$ *is a regular measure of regret that projects to* $\mathcal{R}$, *then* $\mathcal{Q} = \{ Q \in \operatorname{dom} \mathcal{V}^* \mid EQ = 1 \}$ *and the conjugate* $\mathcal{J} = \mathcal{R}^*$ *has* $\mathcal{J}(Q) = \mathcal{V}^*(Q)$ *for* $Q \in \mathcal{Q}$.

The error measure $\mathcal{E}$ *paired with the regret measure* $\mathcal{V}$ *has* $\mathcal{E}^*(X) = \mathcal{V}^*(X + 1)$. *Likewise, the deviation measure* $\mathcal{D}$ *paired with the risk measure* $\mathcal{R}$ *has* $\mathcal{D}^*(X) = \mathcal{R}^*(X + 1)$.

Examples of Risk Envelopes

$$\mathcal{R}(X) = EX + \lambda\sigma(X) \;\longleftrightarrow\; \mathcal{Q} = \left\{ 1 + \lambda Y \;\middle|\; ||Y||_2 \leq 1,\; EY = 0 \right\}$$

$$\mathcal{R}(X) = \mathrm{CVaR}_\alpha(X) \;\longleftrightarrow\; \mathcal{Q} = \left\{ Q \;\middle|\; 0 \leq Q \leq \tfrac{1}{1-\alpha},\; EQ = 1 \right\}$$

$$\mathcal{R}(X) = \sup X \;\longleftrightarrow\; \mathcal{Q} = \left\{ Q \,|\, Q \geq 0,\; EQ = 1 \right\}$$

$$\mathcal{R}(X) = \textstyle\sum_{k=1}^r \lambda_k \mathcal{R}_k(X) \;\longleftrightarrow\; \mathcal{Q} = \left\{ \textstyle\sum_{k=1}^r \lambda_k Q_k \;\middle|\; Q_k \in \mathcal{Q}_k \right\}, \text{ where } \mathcal{R}_k \;\longleftrightarrow\; \mathcal{Q}_k.$$

$$\mathcal{R}(X) = \sum_{k=1}^r p_k \sup_k X \;\longleftrightarrow\; \mathcal{Q} = \left\{ Q \geq 0 \,|\, E[Q|\Omega_k] = p_k \right\}$$

$$\mathcal{R}(X) = \log E[\exp X] \;\longleftrightarrow\; \mathcal{J}(Q) = \begin{cases} E[Q \log Q] & \text{if } Q \geq 0,\; EQ = 1, \\ \infty & \text{otherwise.} \end{cases}$$

Library of Test Problems

- Google: URYASEV
- Go to the first link: University of Florida home page of URYASEV: http://www.ise.ufl.edu/uryasev/
- Go to "Test problems with data and calculation results:" http://www.ise.ufl.edu/uryasev/testproblems/

Hedging Strategies for Equities

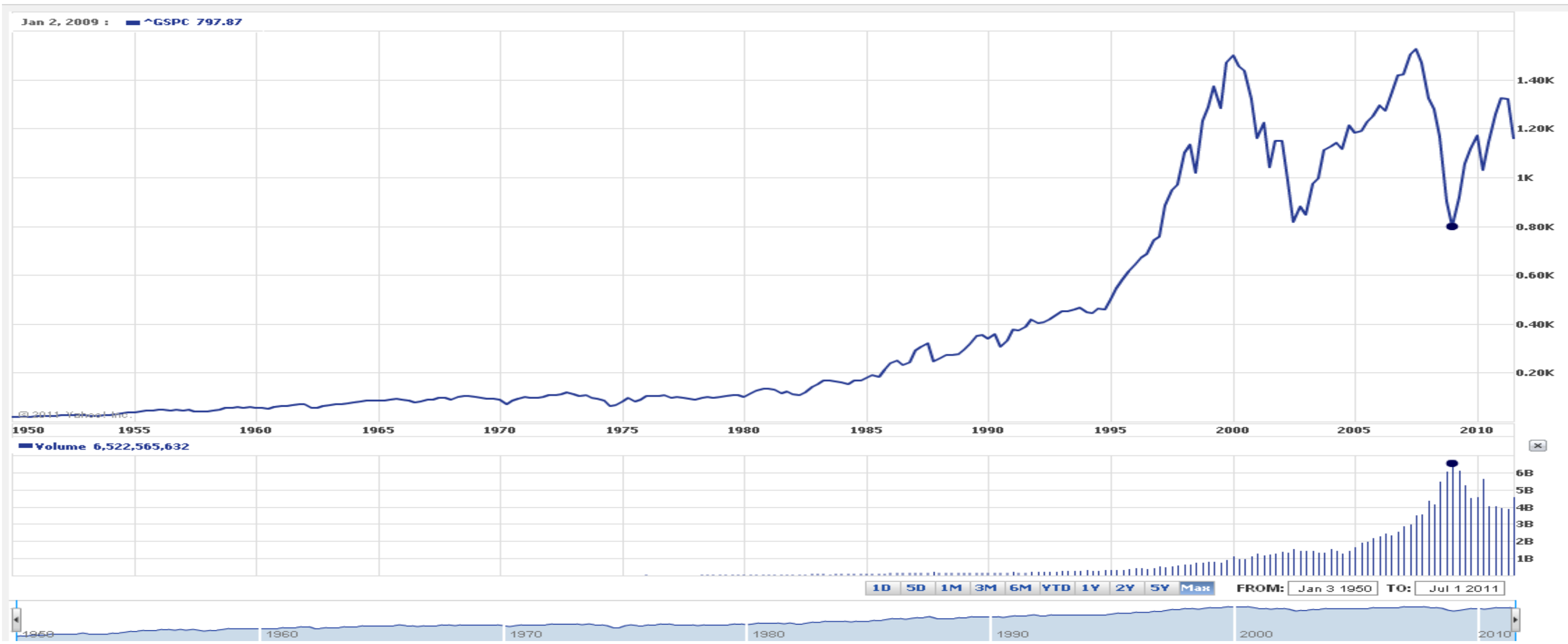
- This part of the presentation is based on paper

  Serraino, G. and S. Uryasev. Protecting Equity Investments: Options, Inverse ETFs, Hedge Funds, and AORDA Portfolios. American Optimal Decisions, Gainesville, FL. March 17, 2011.

  link: www.aorda.com/aod/static/documents/Protecting_Equity_Investments.pdf

- References on cited further papers can be found in Serraino and Uryasev paper

S&P500 01/1950 - 09/2011 (Yahoo Finance)

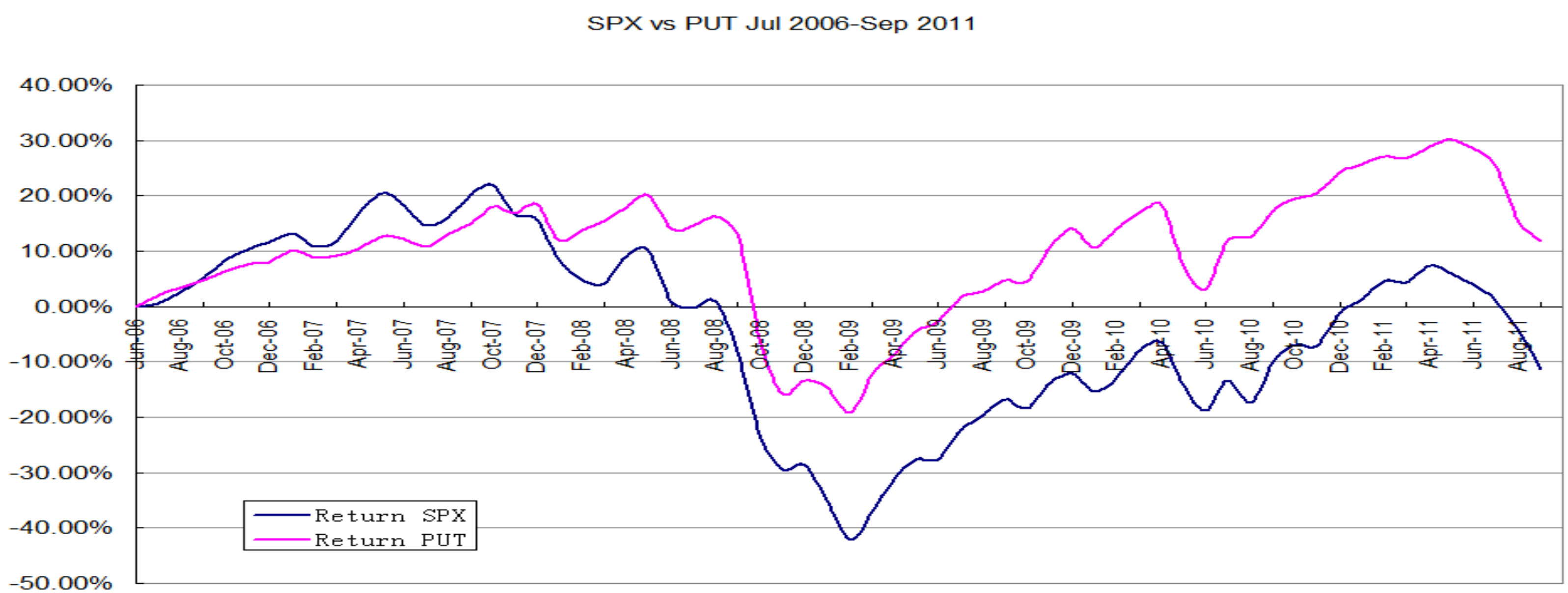


- 12 years of market stagnation: LARGE LOSSES for investors.
  - Assumptions: 2% management fees per year (combined fees of the advisor and mutual funds) + 3% inflation = total loss 5% per year in constant (uninflated) dollars.
  - Total cumulative loss 46% of purchasing power in constant dollars over the recent 12 years,

1-0.95^12= 0.46

Hedging with Put Options and Portfolio Insurance

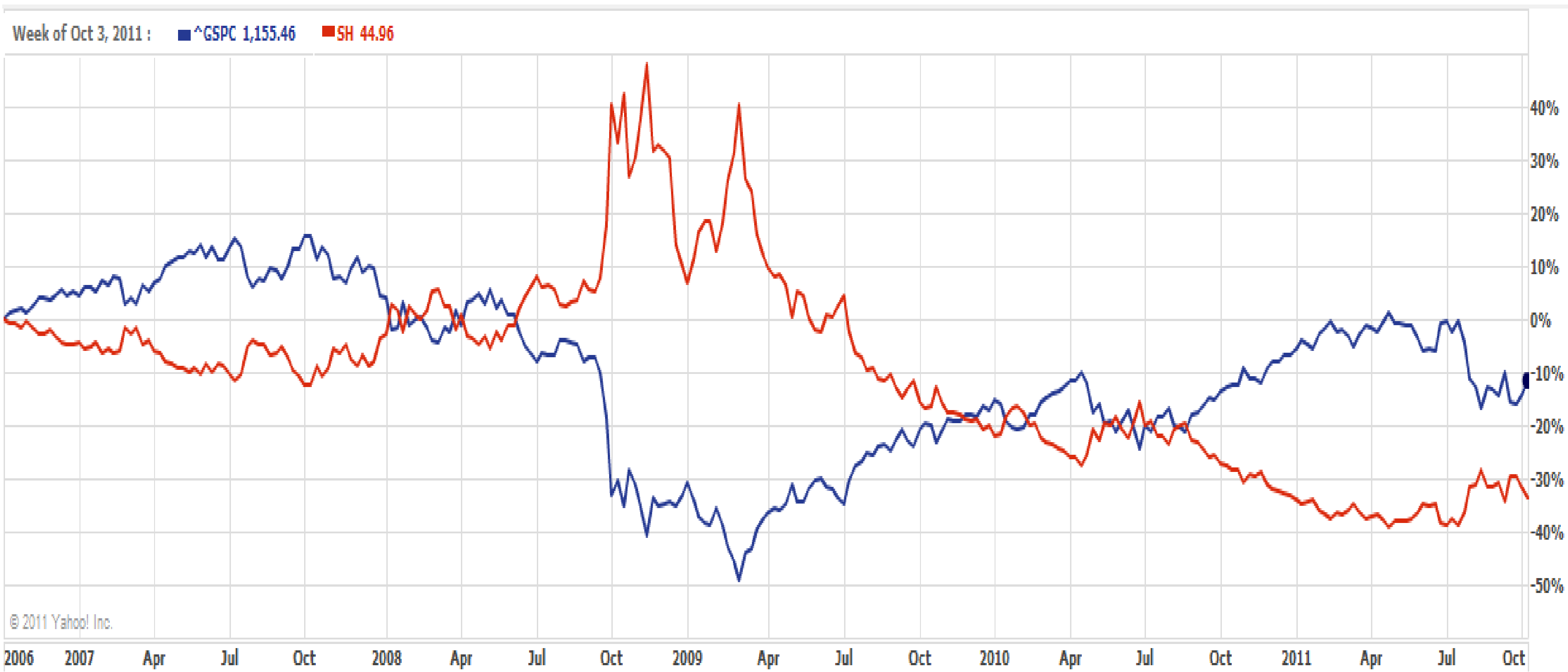
- CBOE PutWrite Index sells at-the-money put options on S&P500 on monthly basis
- (Profits PutWrite) > (Profits S&P500), i.e. S&P500 protection costs more than profits from S&P500. Similar statement is valid for portfolios insurance approaches.



CBOE S&P500 PutWrite Index vs. S&P500. Source: www.cboe.com

Hedging with Inverse ETFs

- Exchange Traded Fund SH provides negative returns of S&P500
- SH is <u>not a good long-term</u> hedge against S&P500 drawdowns



S&P500 vs SH, Jul 2006 –Oct 2011. Yahoo Finance.

Hedge Funds: Positive and Negative Volatility Exposure

- Bondarenko (2004) shows that for most categories of hedge funds a significant fraction of returns can be explained by a negative loading on a volatility factor. i.e., the majority of hedge funds <u>short</u> volatility.
- Lo (2001, 2010) describes a hypothetical hedge fund, "Capital Decimation Partners", shorting out-of-the-money S&P500 put options on monthly basis with strikes approximately 7% out of the money.
- Agarwal and Naik (2004): many hedge fund categories exhibit returns similar to those from selling put options, and have a negative exposure to volatility risk.

Capital Decimation Partners, L.P.

Performance Summary, January 1992 to December 1999

Statistic	S&P 500	CDP
Monthly Mean	1.4%	3.7%
Monthly Std. Dev.	3.6%	5.8%
Min Month	−8.9%	−18.3%
Max Month	14.0%	27.0%
Annual Sharpe Ratio	0.98	1.94
# Negative Months	36/96	6/96
Correlation with S&P 500	100.0%	59.9%
Total Return	367.1%	2721.3%

Capital Decimation Partners, L.P.

Monthly Performance History

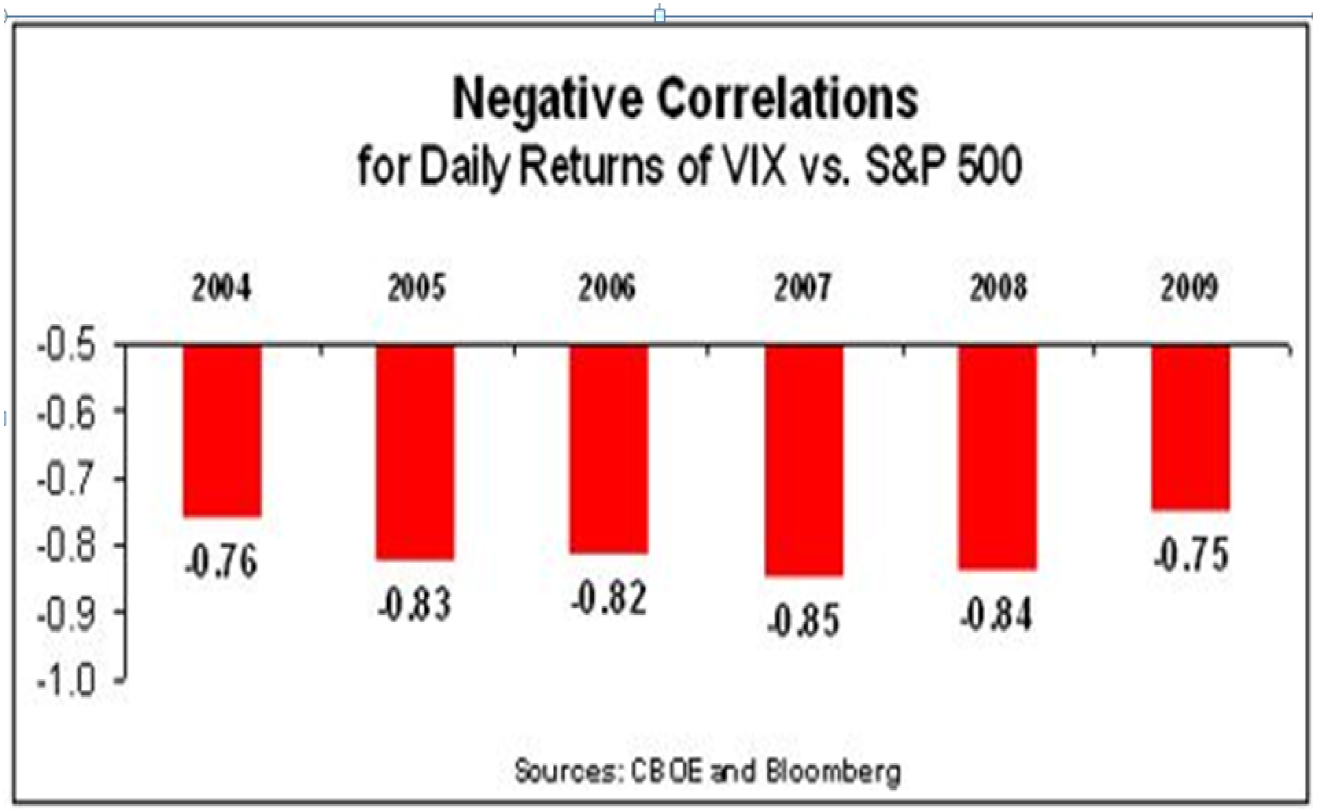
Month	1992		1993		1994		1995		1996		1997		1998		1999	
	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP
Jan	8.2	8.1	−1.2	1.8	1.8	2.3	1.3	3.7	−0.7	1.0	3.6	4.4	1.6	15.3	5.5	10.1
Feb	−1.8	9.3	−0.4	1.0	−1.5	0.7	3.9	0.7	5.9	1.2	3.3	6.0	7.6	11.7	−0.3	16.6
Mar	0.0	4.9	3.7	3.6	0.7	2.2	2.7	1.9	−1.0	0.6	−2.2	3.0	6.3	6.7	4.8	10.0
Apr	1.2	3.2	−0.3	1.6	−5.3	−0.1	2.6	2.4	0.6	3.0	−2.3	2.8	2.1	3.5	1.5	7.2
May	−1.4	1.3	−0.7	1.3	2.0	5.5	2.1	1.6	3.7	4.0	8.3	5.7	−1.2	5.8	0.9	7.2
Jun	−1.6	0.6	−0.5	1.7	0.8	1.5	5.0	1.8	−0.3	2.0	8.3	4.9	−0.7	3.9	0.9	8.6
Jul	3.0	1.9	0.5	1.9	−0.9	0.4	1.5	1.6	−4.2	0.3	1.8	5.5	7.8	7.5	5.7	6.1
Aug	−0.2	1.7	2.3	1.4	2.1	2.9	1.0	1.2	4.1	3.2	−1.6	2.6	−8.9	−18.3	−5.8	−3.1
Sep	1.9	2.0	0.6	0.8	1.6	0.8	4.3	1.3	3.3	3.4	5.5	11.5	−5.7	−16.2	−0.1	8.3
Oct	−2.6	−2.8	2.3	3.0	−1.3	0.9	0.3	1.1	3.5	2.2	−0.7	5.6	3.6	27.0	−6.6	−10.7
Nov	3.6	8.5	−1.5	0.6	−0.7	2.7	2.6	1.4	3.8	3.0	2.0	4.6	10.1	22.8	14.0	14.5
Dec	3.4	1.2	0.8	2.9	−0.6	10.0	2.7	1.5	1.5	2.0	−1.7	6.7	1.3	4.3	−0.1	2.4
Year	14.0	46.9	5.7	23.7	−1.6	33.6	34.3	22.1	21.5	28.9	26.4	84.8	24.5	87.3	20.6	105.7

S&P500 vs VIX

- VIX is implied volatility from prices of options on S&P500 (Jan 2006 – Jan 2011 graph)
- Hedge funds with long volatility exposure provide good hedging protection for investors because they have high returns when the market goes down and when volatility is high.
- Volatility is very volatile (as measured by VIX)

Negative Correlation of VIX and S&P500

- When VIX rises the stock prices fall, and as VIX falls, stock prices rise



Volatility is Very Volatile

- VIX volatility was higher than volatility of VX Near-Term futures, S&P500 (SPX), Nasdaq100 (NDX), Russell 2000 (RUT), stocks, including Google and Apple.

Name	12/31/08 Price	2008 Volatility
VIX	40.00	127.3%
VX Near-Term Futures	41.94	88.9%
SPX	903.25	41.0%
NDX	1,211.65	42.3%
RUT	499.45	46.4%
GOOG	307.65	55.2%
AAPL	85.35	58.2%

Name	12/31/09 Price	2009 Volatility
VIX	21.68	88.9%
VX Near-Term Futures	22.95	69.2%
SPX	1,115.10	27.3%
NDX	1,860.31	26.5%
RUT	625.39	36.2%
GOOG	619.98	30.1%
AAPL	210.73	33.7%

Good Hedge Funds

- <u>Hedge funds with long volatility exposure provide good hedging protection</u> for investors because they have high returns when the market goes down and when volatility is high.
- <u>Dedicated short bias (DSB)</u> hedge funds, for which short selling is the main source of return have positive performance when the markets fall, exhibited extremely strong results during market downturn.
- Connolly and Hutchinson (2010) show that DSB hedge funds are a significant source of diversification for investors and produce statistically significant levels of alpha

AORDA Portfolios at RYDEX

- American Optimal Advisors website http://www.aorda.com/aoa/
- AORDA_Portfolios.pdf can be downloaded from http://www.aorda.com/aoa/static/documents/investments/AORDA_Portfolios.pdf
- AORDA Portfolios invest to S&P500 index and NASDAQ100 index using the index tracking funds at RYDEX Family of Funds
- "Buy low sell high" strategy on daily basis; no positions overnight in the indices.

AORDA Portfolios at RYDEX

- CVaR optimal portfolio

Maximizing expected return

$$\max \; ExpectedReturn(\vec{x})$$

Risk constraint (90%-CVaR is bounded)

$$\text{CVaR}_{90\%}(\vec{x}) \leq w$$

Budget constraint

$$\sum_{i=1}^{I} x_i = U$$

Bounds on exposures

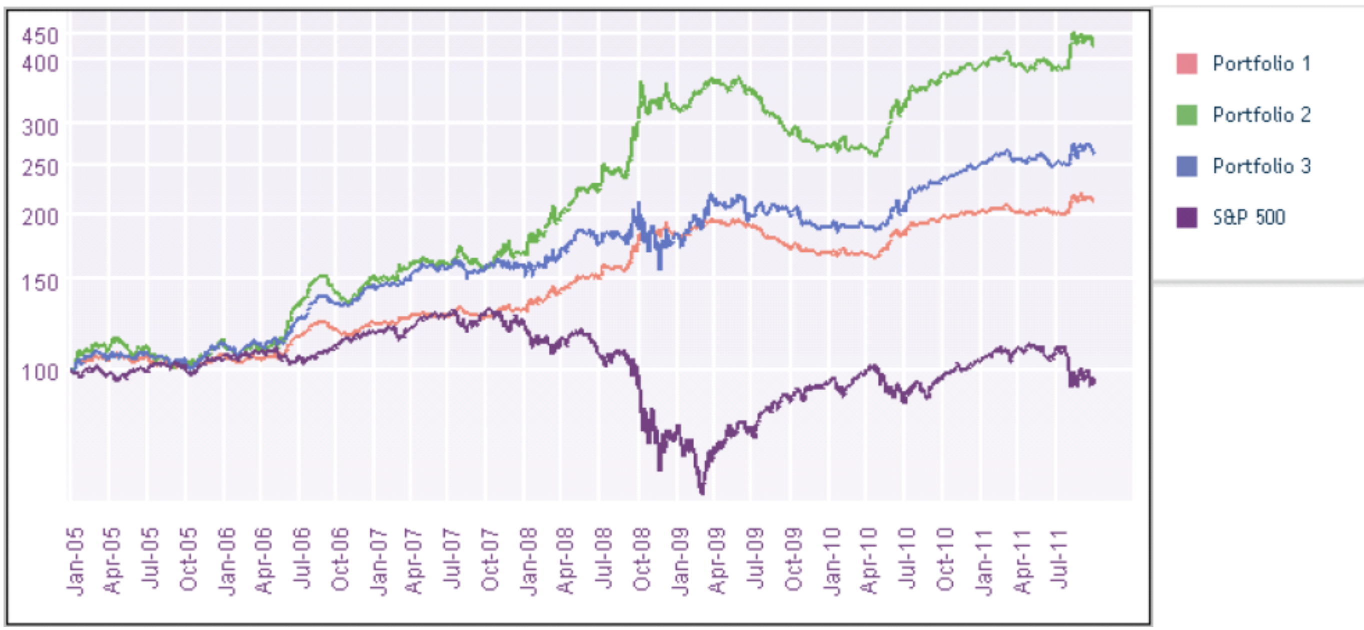
$$l_i \leq x_i \leq u_i$$

where:

I = number of instruments in the portfolio, $i=1,\dots,I$;
$\vec{x}$ = vector of decision variables, i.e., portfolio weights assigned to each instrument;
w = upper bound for CVaR risk;
U = available capital;
l_i = lower bound on exposure to instrument i ;
u_i = upper bound on exposure to instrument i.

AORDA Portfolios at RYDEX (Show aorda_portfolios.pdf)

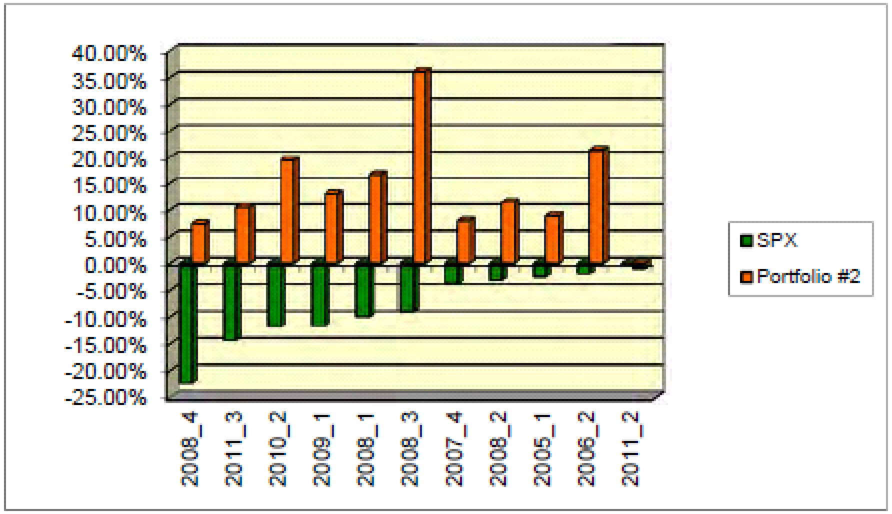
- Portfolio 2 "mirrors" S&P500, and it is negatively correlated with S&P500. On the other hand, Portfolio 2 has a quite high positive return (doubling the value every 3 years). Portfolio 2 has properties of long volatility strategy: it achieves high positive return (exceeding market loss) in bear markets and still attains a positive return (on average) in bull markets. Portfolio 3, which is a mixture of the S&P500 and Portfolio 2, performs quite well both in up and down markets.

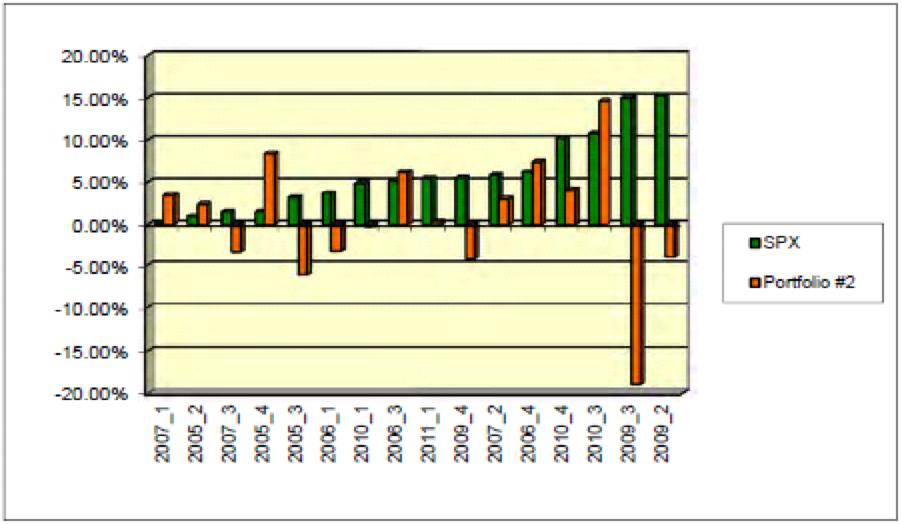


AORDA Portfolios vs. S&P500

AORDA Portfolios at RYDEX (Show www.AORDA.com)

- Left Fig.: negative quarterly returns of S&P500 vs AORDA Portfolio 2 for Jan 2005 - Dec 2010. In all quarters when market return was negative Portfolio 2 had a positive return.
- Right Fig.: positive quarterly returns of S&P500 vs AORDA Portfolio 2 for Jan 2005 - Sep 2011. When the market is up, portfolio 2 had slightly positive return on average. However, Portfolio 2 has tendency to lose when the market has especially high returns.





Portfolio #2 average = 13.86%,
S&P500 (SPX) average = -8.29%

Portfolio #2 average = 0.61%,
S&P500 (SPX) average = 5.95%

Trading Track Record of AORDA Portfolios

	Portfolio #1	Portfolio #2	Portfolio #3	S&P 500 (SPX)
Since Inception	111.77	324.80	162.74	-6.64
Inception/Annual	11.76	23.90	15.39	-1.01
2011 (Jan-Sep)	5.10	9.91	5.68	-10.04
2010	19.61	42.15	31.22	12.78
2009	-8.08	-15.96	3.40	23.45
2008	39.63	90.56	15.15	-38.49
2007	5.89	11.46	8.03	3.53
2006	16.06	34.15	32.00	13.62
2005	6.81	13.54	11.60	3.00

Performance Summary of AORDA Portfolios (Cont'd)

PERFORMANCE CATEGORY	Portfolio #1	Portfolio #2	Portfolio #3	S&P 500 (SPX)	NDX	DJI
Cumulative Return (%)	111.77	324.80	162.74	-6.64	31.96	1.21
Annual Compounded Rate of Return (%)	11.76	23.90	15.39	-1.01	4.19	0.18
Sharpe Ratio (Risk Free = 0%)	1.16	1.15	1.10	0.02	0.31	0.09
Sortino Ratio (Risk Free = 0%)	2.72	2.75	2.08	0.02	0.43	0.12
Correlation with S&P500 (SPX) (%)	-37.85	-37.01	18.16	100	89.68	97.54
Maximum Portfolio Drawdown (%)	15.06	28.16	14.75	52.56	50.11	49.30
Annual Standard Deviation (%)	10.03	20.45	13.92	16.15	20.08	14.98
Annual α-coefficient (%)	11.72	23.71	15.29	n/a	5.81	1.04

Balanced Portfolios

PORTFOLIO PERFORMANCE CATEGORY	100% in S&P500 (SPX) and 0% in Portfolio #2	75% in S&P500 (SPX) and 25% in Portfolio #2	50% in S&P500 (SPX) and 50% in Portfolio #2
Annualized ROR (Compounded) (%)	-1.01	5.63	12.03
Annualized Std. Deviation (%)	16.15	11.27	10.42
Sortino Ratio	0.02	0.75	2.39
Sharpe Ratio	0.02	0.54	1.15
Largest Drawdown (%)	52.56	29.73	13.52

PORTFOLIO PERFORMANCE CATEGORY	100% in S&P500 (SPX) and 0% in Portfolio #3	75% in S&P500 (SPX) and 25% in Portfolio #3	50% in S&P500 (SPX) and 50% in Portfolio #3
Annualized ROR (Compounded) (%)	-1.01	3.24	7.40
Annualized Std. Deviation (%)	16.15	13.20	11.58
Sortino Ratio	0.02	0.39	1.03
Sharpe Ratio	0.02	0.31	0.68
Largest Drawdown (%)	52.56	39.91	24.92